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(i) The magnitude of the gravitational force experienced by the mass $m$ at a depth $h$ from the surface (see figure) will be
(A) $\quad m g\left(1-\frac{h}{R}\right)$
(B) $m g\left(1+\frac{h}{R}\right)$
(C) $m g \frac{h}{R}$
(D) $m g \frac{h}{R-h}$

Correct answer: (A)
$g(r)=G M(r) / r^{2} \quad M(r)=(4 / 3) \rho \pi r^{3} \quad \rho=M /(4 / 3) \rho \pi R^{3}$
$g(r)=\left(G M / R^{2}\right)(r / R)=g r / R \quad F=[m g / R] r=m g(1-h / R)$
(ii) In the grid provided on your answer sheet, plot a graph of $F(r) / m g$, where $F(r)$ is the force on the mass $m$ at a distance $r$ from the centre of the Moon, as a function of $r / R$, as $r$ varies from 0 to $2 R$.

(iii) If $m=0.10 \mathrm{~kg}$, what is the minimum time (in seconds) it will take, from the moment the mass $m$ is dropped through the hole at the surface, for it to reach the centre of the Moon?
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Correct answer: $\mathrm{T}=1619 \mathrm{~s}$
Substitute $\mathrm{mg} / \mathrm{R}$ as $k$ in the formula for the frequency, Time period $=6476$ s , time take to reach the centre $=1619 \mathrm{~s}$.
$v_{0}=1 / 2 \pi(k / m)^{0.5}$
Time taken to reach the centre, $T=2 \pi / 4(\mathrm{~m} / \mathrm{k})^{0.5}$

$$
T=\pi / 2(R / g)^{0.5} \quad T=1619 \mathrm{~s}
$$

(iv) The oxygen molecule $\mathrm{O}_{2}$ has a force constant, $k=1150 \mathrm{~N} \mathrm{~m}^{-1}$. The equilibrium bond length is $L=1.5 \times 10^{-10} \mathrm{~m}$ and the change in the bond length when it is fully stretched is $6.0 \%$ of $L$. Calculate the vibrational energy, that is the sum of kinetic and potential energies per mole of oxygen (in $\mathrm{kJ} \mathrm{mol}^{-1}$ ). [1.5]
(Avogadro's number, $N_{A}=6.023 \times 10^{23}$ )
Vibrational energy per mole $=U N_{0}=28.052 \mathrm{~kJ} / \mathrm{mole}$
$L_{s}=1.59 \times 10^{-10} \mathrm{~m} \quad[0.5]$
Vibration energy is the work done in changing the separation from equilibrium position to the fully stretched position [1.0].
Force, $F=k\left(L_{s}-L\right) \quad$ Work, $W=0.5 k\left(L_{s}-L\right)^{2}$
$U=0.5 k\left(L_{s}-L\right)^{2}$
Vibrational energy per mole $=U N_{0}=28.052 \mathrm{~kJ} / \mathrm{mole}$
(v) The atomic weights of the halogen elements listed in the periodic table are:

| $\mathbf{F}$ | $\mathbf{C l}$ | $\mathbf{B r}$ | $\mathbf{I}$ |
| :---: | :---: | :---: | :---: |
| 19.0 | 35.5 | 79.9 | 126.9 | International Junior Science Olympiad,

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Two halogen elements, X and Y , form diatomic molecules $\mathrm{X}_{2}$ and $\mathrm{Y}_{2}$ with force constants $k_{X}=325.0 \mathrm{~N} \mathrm{~m}^{-1}$ and $k_{Y}=446.0 \mathrm{~N} \mathrm{~m}^{-1}$ respectively. The vibration frequencies are measured to be $v_{X}=16.7 \times 10^{12} \mathrm{~Hz}$ and $v_{Y}=26.8 \times 10^{12} \mathrm{~Hz}$. Identify the halogen elements X and Y by writing their symbols. Write your answer in the form $\mathrm{X}=$ $\qquad$ , $\mathrm{Y}=$ $\qquad$ in the answer sheets.

Correct answer: $\mathrm{X}=\mathrm{Cl}$ and $\mathrm{Y}=\mathrm{F}$
The ratio of the vibration frequencies is 0.623 . Only $\mathrm{X}=\mathrm{Cl}$ and $\mathrm{Y}=\mathrm{F}$ are the elements such that the ratio of the square roots of the masses $\left(m_{y} / m_{x}\right)^{0.5} \sim 0.64$.

## Question 2

(i) If sunlight is shone through a transparent container (with walls of negligible thickness) filled with nitrogen gas, what will be the ratio of the scattered light intensity for colours corresponding to wavelengths 400 nm and 650 nm respectively?

Ratio of the scattered intensities $=6.2$
Correct ratio of the light intensity $=0.9 \quad[0.5]$ or
Correct ratio of the scattered light intensity without taking into account solar spectrum [0.5].

The ratio of the intensity of 400 nm and 650 nm light in the sunlight is 0.9 .
$\eta_{s}(400) / \eta_{s}(650)=(650 / 400)^{4 *} 0.9=6.2$
[0.5].

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Scattering losses due to the pollutants will be proportional to $0.1^{*} 40=4$ times larger compared to pure air.

Pollutants act as additional scattering hence the visibility range reduces by 5 times ( $\mathbf{6 0 \times 1 0 ^ { 3 }} \mathbf{~ m}$ )
(ii) Milk is a colloidal solution in which droplets of liquid fat, of size around 100 nm , are suspended in water. These droplets scatter light more strongly than the water molecules, causing normal milk to appear white rather than transparent.

Consider the following experiment. A few drops of milk are added to a glass of water illuminated from above by
 a beam of sunlight, as shown in the figure on the right. The water turns cloudy, but some sunlight still passes through, since the concentration of milk is small. The glass is now viewed (I) from below, and (II) from the side, as shown in the figure.

When compared to the emerging light viewed from below (I), the emerging light viewed from the side (II) will appear
(A) bluish
(B) orange
(C) reddish
(D) the same

## Correct answer: (A)

Shorter wavelengths are scattered much more than the longer wavelengths, hence blue will be scattered more along the direction (II) compared to direction (I)
(iii) Which of the following atmospheric phenomena is mainly governed by Mie scattering of light?
[0.5]

(A) red sunset

(C) blue sky

(B) white clouds

(D) rainbow

Images taken from:
(A) http://bostern.wordpress.com
(B) http://www.kaneva.com
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[^0]Correct answer: (B) white clouds
(a) and (c) are due to Rayleigh scattering (wavelength dependent), (d) is due to wavelength dependent refraction and dispersion through water droplets.
(i) The green colour of leaves and shoots of plants is usually due to the presence of chlorophyll, the compound mainly responsible for photosynthesis. Which of the following graphs depicts the correct absorbance spectrum of chlorophyll?

## Correct answer: (B)

For green colour the absorption is minimum and hence is reflected or scattered the most
(ii) Assuming that the rate of photosynthesis is proportional to the amount of light absorbed (see above figure), what will be the wavelength (in nm ) corresponding to the maximum photosynthesis rate in green plants?

Maximum absorption in Fig. $b$ is at 450 nm .
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Larger absorbance where the solar spectrum has a maximum intensity.

## Question 3

(a) The dissociation of $\mathrm{H}_{2} \mathrm{CO}_{3}$ in blood occurs in two steps. Write down balanced equations for these two steps.

$$
\begin{align*}
& \mathrm{H}_{2} \mathrm{CO}_{3}{ }_{\text {(aq.) }} \rightleftharpoons \mathrm{H}^{+}{ }_{\text {(aq. })}+\mathrm{HCO}_{3}^{-}{ }_{(\text {aq. })}  \tag{0.25}\\
& \mathrm{HCO}_{3}{ }^{-}{ }_{\text {(aq.) }} \rightleftharpoons \mathrm{H}^{+}{ }_{\text {(aq.) }}+\mathrm{CO}_{3}^{2-}{ }_{(\text {aq. })} \tag{0.25}
\end{align*}
$$

(i) Calculate the concentration of $\mathrm{H}^{+}$in a solution at $37{ }^{\circ} \mathrm{C}$, and hence its pH value, if $\mathrm{H}_{2} \mathrm{CO}_{3}$ and $\mathrm{HCO}_{3}^{-}$are present in equal concentrations in $\mathrm{mol} / \mathrm{l}$ in that solution.

$$
\begin{array}{ll}
{\left[\mathrm{H}_{2} \mathrm{CO}_{3}\right]=\left[\mathrm{HCO}_{3}^{-}\right]} & \quad \mathrm{K}_{1}=\left[\mathrm{H}^{+}\right] \\
{\left[\mathbf{H}^{+}\right]=\mathbf{2 . 2} \times 10^{-4} \mathbf{M}} & {[\mathbf{0 . 2 5 ]}} \\
\mathbf{p H}=\mathbf{3 . 6 5} & {[\mathbf{0 . 2 5 ]}} \tag{0.25}
\end{array}
$$

(ii)

$$
\begin{gather*}
\mathrm{pH}=7.4 \Rightarrow\left[\mathrm{H}^{+}\right]=3.98 \times 10^{-8}  \tag{0.5}\\
K_{2}=\frac{\left[\mathrm{H}^{+}\right]\left[\mathrm{CO}_{3}^{-2}\right]}{\left[\mathrm{HCO}_{3}^{-}\right]}=4.8 \times 10^{-11}=\frac{3.98 \times 10^{-8}\left[\mathrm{CO}_{3}^{-2}\right]}{\left[\mathrm{HCO}_{3}^{-}\right]} \\
\frac{\left[\mathrm{CO}_{3}^{-2}\right]}{\left[\mathrm{HCO}_{3}^{-}\right]}=\frac{4.8 \times 10^{-11}}{3.98 \times 10^{-8}}=1.2 \times 10^{-3} \\
\therefore \frac{\left[\mathrm{HCO}_{3}^{-}\right]}{\left[\mathrm{CO}_{3}^{-2}\right]}=\frac{1}{1.2 \times 10^{-8}}=833 \tag{0.5}
\end{gather*}
$$

(b) Usually in the human body, $\mathrm{H}_{2} \mathrm{CO}_{3}$ is in equilibrium with the $\mathrm{CO}_{2}$ dissolved in the blood. $K_{3}$

$$
\mathrm{CO}_{2} \text { (dissolved) }+\mathrm{H}_{2} \mathrm{O} \text { (liq) } \rightleftarrows \mathrm{H}_{2} \mathrm{CO}_{3} \text { (dissolved) }
$$

$$
\text { At } 37^{0} \mathrm{C}, K_{3}=5.0 \times 10^{-3}
$$

Calculate the total equilibrium constant, $K^{\prime}$ for the reaction

$$
\begin{gather*}
\mathrm{CO}_{2} \text { (dissolved) }+\mathrm{H}_{2} \mathrm{O} \text { (liq) } \rightleftarrows \mathrm{HCO}_{3}^{-}(\mathrm{aq})+\mathrm{H}^{+}(\mathrm{aq}) \\
K_{1}=\frac{\left[\mathrm{H}^{+}\right]\left[\mathrm{HCO}_{3}^{-2}\right]}{\left[\mathrm{H}_{2} \mathrm{CO}_{3}\right]} \\
K^{f}=\frac{\left[\mathrm{H}^{+}\right]\left[\mathrm{HCO}_{3}^{-2}\right]}{\left[\mathrm{CO}_{2}\right]} \\
\\
\left(5.0 \times 10^{-3}\right)\left(2.2 \times 10^{-4}\right)=1.1 \times 10^{-6} \tag{0.5}
\end{gather*}
$$

(c) Blood plasma contains a total carbonate buffer pool, which is a mixture of $\mathrm{HCO}_{3}^{-}$and $\mathrm{CO}_{2}$ with a total concentration of $3.4 \times 10^{-2} \mathrm{M}$ at $38^{0} \mathrm{C}$. At this temperature the value of the equilibrium constant $K^{\prime}$ is $1.3 \times 10^{-6}$. The concentration of the $\mathrm{H}_{2} \mathrm{CO}_{3}$ is
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negligible. Calculate the ratio of concentrations of $\mathrm{CO}_{2}$ (dissolved) and $\mathrm{HCO}_{3}^{-}$, and their individual concentrations, in this blood sample at pH 7.4 .

$$
\begin{gathered}
\mathrm{pH}=7.4 \Rightarrow\left[\mathrm{H}^{+}\right]=3.98 \times 10^{-8} \\
\mathrm{~K}_{2}=\frac{\left[\mathrm{H}^{+}\right]\left[\mathrm{HCO}_{3}^{-}\right]}{\left[\mathrm{CO}_{2}\right]}=\frac{1.3 \times 10^{-6}}{3.98 \times 10^{-8}} \\
\left.\frac{[\mathrm{HCO}}{3}\right] \\
{\left[\mathrm{CO}_{2}\right]}
\end{gathered}=32.6 \quad \text { or } \quad \frac{\left[\mathrm{CO}_{2}\right]}{\left[\mathrm{HCO}_{3}^{-}\right]}=0.0306 .\left[\mathrm{CO}_{2}\right]=3.4 \times 10^{-2} \mathrm{32.6}\left[\mathrm{CO}_{2}\right]+\left[0^{-2} .\right.
$$

$$
\text { Question } 4
$$

(a) Which of the above carry de-oxygenated blood?
(b)

What would be the individual's heart rate (in beats/minute) as calculated from the table?
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Heart rate: $60 / 0.7=85.7$ beats/min
(c)

| Time | Mitral valve | Aortic valve |
| :--- | :--- | :--- |
| 0.2 s | C | O |
| 0.6 s | O | C |

(d) Blood flows from the heart into the aorta during a cardiac cycle. If the diameter of the aorta is approximately 2.4 cm , then, using the table in 4 (b) calculate the average speed (in $\mathrm{cm} \mathrm{s}^{-1}$ ) of blood flowing into the aorta in one full cardiac cycle.

Speed $=$ Volume/(cycle time*Area)
Speed $=42 / 4.52=60 / 4.52$
Speed $=13.26 \mathrm{~cm} / \mathrm{s}$ (for cycle time of 0.7 s ) or $23.23 \mathrm{~cm} / \mathrm{s}$ (for cycle time of 0.4 s ) [0.5]
(e) Blood flows from the aorta and its major arteries into arterioles and fine-walled capillaries. If all the major arteries in the body have a total cross-sectional area of about $7.0 \mathrm{~cm}^{2}$ calculate the average speed (in $\mathrm{cm}-\mathrm{s}^{-1}$ ) in the major arteries which have the same volume of blood as the aorta flowing through them.
$\mathrm{V} 2=8.56 \mathrm{~cm} / \mathrm{s}(13.26 \mathrm{~cm} / \mathrm{s}), \mathrm{V} 2=14.99 \mathrm{~cm} / \mathrm{s}(23.23 \mathrm{~cm} / \mathrm{s})$
A1V1 = A2V2

FULL [0.5] Credits to be given for the correct value.
HALF [0.25] Credits to be given for the formula even if the value is wrong.
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| Aorta | Renal <br> vein | Alveolar <br> space in <br> lungs | Pulmonary <br> artery |
| :---: | :---: | :---: | :---: |
| Y | X | Y | X |

## Question 5

(i) Calculate the average acceleration of this cheetah required to reach its maximum speed.

Average acceleration, $a=\frac{v}{t}$
$\mathrm{a}=\frac{30}{3}=10 \mathrm{~ms}^{-2}$
(ii) Calculate the distance travelled during the first 3.0 s , assuming that the acceleration is uniform.

Distance travelled during the acceleration phase, $d^{1}=\frac{1}{2} a t^{2}$
$d^{1}=\frac{1}{2} \times 10 \times 3^{2}=45 \mathrm{~m}$
(iii) The cheetah has to do work against friction, mostly due to air. Assume that this frictional force is always 100 N . Calculate the total mechanical work done by the
cheetah during the first 23.0 s of its motion.

Work done in
Change in kinetic energy, $K=\frac{1}{2} m v^{2}-0$
$K=\frac{1}{2} \times 50 \times 30^{2}=22.5 \mathrm{~kJ}$
Overcoming friction due to air, $W_{d}=F_{a} \times\left(d+d^{1}\right)$
$W_{d u}=100 \times(600+45)=64.5 \mathrm{~kJ}$
Total work done $W=K+W_{d}$
$W=22.5+64.5=87 \mathrm{~kJ}$
(a) During the first 23.0 s , the body temperature of the cheetah rises from $38.5^{\circ} \mathrm{C}$ to
${ }^{0} \mathrm{C}$. Take the specific heat of the body of the cheetah to be $4.2 \mathrm{~kJ} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$.
(vi) If the rise in body temperature is linear during this time, calculate the total heat generated by the cheetah's metabolism. Neglect any heat loss to the surroundings.
[1.0]
Heat generated in the body of the Cheetah, $H=\operatorname{Sm\Delta t}$

$$
\begin{equation*}
H=4.2 \times 50 \times(40-38.5)=315 \mathrm{~kJ} \tag{0.5}
\end{equation*}
$$

(vii) Assume that some of the energy generated by the cheetah's body increases its temperature and the rest corresponds to the mechanical work done. Calculate the fraction of the total generated energy that is converted to kinetic energy.

Total energy generated in the body of Cheetah, $E=H+W$

$$
\begin{equation*}
E=315+87=402 k J \tag{0.5}
\end{equation*}
$$

$$
\begin{equation*}
\text { Fraction }=\frac{22.5}{402}=0.06 \tag{0.5}
\end{equation*}
$$

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(viii) Write down the balanced chemical reaction for aerobic respiration.

The balanced reaction: $\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}+6 \mathrm{O}_{2} \longrightarrow 6 \mathrm{CO}_{2}+6 \mathrm{H}_{2} \mathrm{O}$
(ix) If the cheetah requires 400 kJ of energy, calculate the volume of oxygen required if all this energy is to be obtained by aerobic respiration. Take the molar volume of oxygen gas to be 24.5 litres.

Number of glucose molecules required to produce 400 kJ of energy is

$$
\begin{align*}
& \quad n_{g}=\frac{400 \mathrm{~kJ}}{U} \\
& n_{g}=\frac{400 \mathrm{~kJ}}{U 1130 \mathrm{~kJ}}=0.35 \mathrm{~mol}  \tag{0.25}\\
& \text { Number of mol of } O_{2} \text { is } n_{O}=6 n_{g} \\
& n_{o}=6 \times 0.35=2.1 \mathrm{~mol}  \tag{0.25}\\
& \text { Volume of } O_{2} \text { required, } V=24.5 \times n_{O}=24.5 \times 2.1=52 \mathrm{l} \tag{0.5}
\end{align*}
$$

(x) The cheetah extracts oxygen from the air while breathing. The inhaled air (about 500 ml per breath) contains $20.0 \%$ oxygen (by volume), while the exhaled air is assumed to contain $15.0 \%$ oxygen (by volume). Calculate the volume of oxygen that the cheetah can use during the 23.0 s of its run, at a breathing rate of 150 breaths per minute.

Amount of $\mathrm{O}_{2}$ absorbed by the lungs per breath is $V^{1}=5 \%$ of $500 \mathrm{ml}=25 \mathrm{ml}$

Total number of breaths during its entire motion,
$n=\frac{N}{60} \times$ total duration of motion
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$$
\begin{equation*}
n=\frac{150}{60} \times 23=57.5 \approx 58 \tag{0.25}
\end{equation*}
$$

Thus the amount of intake of $O_{2}, V_{a}=n \times V^{1}=58 \times 25=1.45 l$
(xi) Anaerobic respiration converts the energy from glucose into ATP. If glucose were to be completely burnt up, one mole would release 2872 kJ of energy. What is the efficiency of anaerobic respiration compared to complete combustion of glucose?

Energy generated in production of one mol of ATP due to oxidation of glucose is
$U^{1}=\frac{U}{36}=\frac{1130}{36}=31.4 \mathrm{~kJ}$
Efficiency of the anaerobic respiration, $\eta=\frac{U_{g}}{U^{1}}$
$\eta=\frac{2 \times 31.4}{2872}=0.022$
(xii) If all the 400 kJ required by the cheetah for its run were to be produced by anaerobic respiration, calculate the total amount of glucose (in kg ) that would be required.

Amount of glucose of required, $\quad N=\frac{400}{2 U_{g}}$
$N=\frac{400}{62.8}=6.4 \mathrm{~mol}$
Approximate molecular mass of glucose,
$M=6 \times 12+12 \times 1+6 \times 16=180 g$
$m=180 \times 6.4=1.2 \mathrm{~kg}$


[^0]:    (C) http://lisathatcher.wordpress.com
    (D) http://www.freefoto.com

